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РАСЧЕТНЫЕ ЗАДАНИЯ ПО МАТЕМАТИКЕ. УРАВНЕНИЯ МАТЕМАТИЧЕСКОЙ ФИЗИКИ.

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Представленное пособие предназначено для студентов третьего курса физического факультета и является естественным дополнением учебного пособия тех же авторов "Уравнения математической физики. Сборник задач". Подбор задач в данном расчетном задании тематически соответствует этому сборнику. Он содержит 25 вариантов заданий; каждый вариант содержит 12 задач. Вследствие дефицита времени, отведенного для изучения данного раздела математики, а так же достаточно большого объёма вычислительной работы, необходимой для решения задач этого курса, на практических занятиях не всегда возможно охватить весь круг вопросов, с которыми было бы желательно ознакомить студентов. Поэтому самостоятельная работа студентов в этой ситуации оказывается необходимой и чрезвычайно полезной.

Расчетные задания по уравнениям математической физики.

Задача 1. Привести уравнение к каноническому виду и найти его общее решение.

$$1.1\ \ u_{xx}+2u_{xy}+u_{yy}+u_x+u_y=0.$$

$$1.2\ \ u_{xx}+4u_{xy}+4u_{yy}-u_x-2u_y=0.$$

$$1.3\ \ u_{xx}-2u_{xy}+u_{yy}+2u_x-2u_y=0.$$

$$1.4\ \ u_{xx}+6u_{xy}+9u_{yy}+u_x+3u_y=0.$$

$$1.5\ \ u_{xx}-6u_{xy}+9u_{yy}-2u_x+6u_y=0.$$

$$1.6\ \ u_{xx}+2u_{xy}+u_{yy}-3u_x-3u_y=0.$$

$$1.7\ \ u_{xx}-4u_{xy}+4u_{yy}+3u_x-6u_y=0.$$

$$1.8\ \ 9u_{xx}+6u_{xy}+u_{yy}-9u_x-3u_y=0.$$

$$1.9\ \ u_{xx}+8u_{xy}+16u_{yy}-u_x-4u_y=0.$$

$$1.10\ \ u_{xx}-2u_{xy}+u_{yy}+4u_x-4u_y=0.$$

$$1.11\ \ 16u_{xx}+8u_{xy}+u_{yy}-8u_x-2u_y=0.$$

$$41.12\ \ 4u_{xx}+4u_{xy}+u_{yy}+8u_x+4u_y=0.$$

$$1.13\ \ u_{xx}-8u_{xy}+16u_{yy}+3u_x-12u_y=0.$$

$$1.14\ \ 9u_{xx}+6u_{xy}+u_{yy}-12u_x-4u_y=0.$$

$$1.15\ \ 16u_{xx}+8u_{xy}+u_{yy}+16u_x+4u_y=0.$$

$$1.16\ \ u_{xx}+10u_{xy}+25u_{yy}+u_x+5u_y=0.$$

$$1.17\ \ u_{xx}+2u_{xy}+u_{yy}+5u_x+5u_y=0.$$

1.18 $u_{xx} - 10u_{xy} + 25u_{yy} + 2u_x - 10u_y = 0.$

1.19 $4u_{xx} - 4u_{xy} + u_{yy} - 10u_x + 5u_y = 0.$

1.20 $25u_{xx} - 10u_{xy} + u_{yy} - 15u_x + 3u_y = 0.$

1.21 $u_{xx} + 6u_{xy} + 9u_{yy} + 5u_x + 15u_y = 0.$

1.22 $25u_{xx} + 10u_{xy} + u_{yy} + 20u_x + 4u_y = 0.$

1.23 $u_{xx} + 8u_{xy} + 16u_{yy} + 5u_x + 20u_y = 0.$

1.24 $u_{xx} - 10u_{xy} + 25u_{yy} + 5u_x - 25u_y = 0.$

1.25 $u_{xx} + 12u_{xy} + 36u_{yy} + u_x + 6u_y = 0.$

Задача 2. Методом Фурье решить однородное уравнение теплопроводности с нулевыми граничными условиями.

2.1 $u_t = 2u_{xx};\ u(x,0) = \sin 3\pi x;\ \ u(0,t) = u(8,t) = 0.$

2.2 $u_t = 9u_{xx};\ u(x,0) = 2\sin 2\pi x + 3\sin 3\pi x;\ \ u(0,t) = u(1,t) = 0.$

2.3 $u_t = 3u_{xx};\ u(x,0) = 3\sin 2\pi x;\ u(0,t) = u(7,t) = 0.$

2.4 $u_t = 2u_{xx};\ u(x,0) = 4\sin 3\pi x + 5\sin 4\pi x;\ \ u(0,t) = u(2,t) = 0.$

2.5 $u_t = 4u_{xx};\ u(x,0) = 5\sin 3\pi x;\ u(0,t) = u(6,t) = 0.$

2.6 $u_t = 7u_{xx};\ u(x,0) = 6\sin 2\pi x + 7\sin 3\pi x;\ \ u(0,t) = u(3,t) = 0.$

2.7 $u_t = 5u_{xx};\ u(x,0) = 7\sin 2\pi x;\ u(0,t) = u(5,t) = 0.$

2.8 $u_t = 6u_{xx};\ u(x,0) = 8\sin 3\pi x + 9\sin 4\pi x;\ \ u(0,t) = u(4,t) = 0.$

2.9 $u_t = 6u_{xx};\ u(x,0) = 9\sin 3\pi x;\ u(0,t) = u(4,t) = 0.$

2.10 $u_t = 5u_{xx};\ u(x,0) = 10\sin 2\pi x + 3\sin 3\pi x;\ \ u(0,t) = u(5,t) = 0.$

2.11 $u_t = 7u_{xx};\ u(x,0) = 11\sin 2\pi x;\ u(0,t) = u(3,t) = 0.$

2.12 $u_t = 4u_{xx};\ u(x,0) = 12\sin 3\pi x + 5\sin 4\pi x;\ \ u(0,t) = u(6,t) = 0.$

2.13 $u_t = 8u_{xx};\ u(x,0) = 13\sin 3\pi x;\ u(0,t) = u(2,t) = 0.$

2.14 $u_t = 3u_{xx};\ u(x,0) = 14\sin 2\pi x + 7\sin 3\pi x;\ \ u(0,t) = u(7,t) = 0.$

2.15 $u_t = 9u_{xx};\ u(x,0) = 15\sin 2\pi x;\ u(0,t) = u(1,t) = 0.$

2.16 $u_t = 2u_{xx};\ u(x,0) = 16\sin 3\pi x + 9\sin 4\pi x;\ \ u(0,t) = u(8,t) = 0.$

2.17 $u_t = 2u_{xx};\ u(x,0) = 17\sin 2\pi x;\ u(0,t) = u(2,t) = 0.$

2.18 $u_t = 3u_{xx};\ u(x,0) = 18\sin 3\pi x + 3\sin 4\pi x;\ \ u(0,t) = u(7,t) = 0.$

2.19 $u_t = 3u_{xx};\ u(x,0) = 19\sin 3\pi x;\ u(0,t) = u(3,t) = 0.$

2.20 $u_t = 8u_{xx};\ u(x,0) = 20\sin 2\pi x + 7\sin 3\pi x;\ \ u(0,t) = u(6,t) = 0.$

2.21 $u_t = 4u_{xx};\ u(x,0) = 21\sin 3\pi x;\ u(0,t) = u(4,t) = 0.$

2.22 $u_t = 4u_{xx};\ u(x,0) = 22\sin 3\pi x + 5\sin 4\pi x;\ \ u(0,t) = u(5,t) = 0.$

2.23 $u_t = 5u_{xx};\ u(x,0) = 23\sin 3\pi x;\ u(0,t) = u(5,t) = 0.$

2.24 $u_t = 6u_{xx};\ u(x,0) = 24\sin 2\pi x + 9\sin 3\pi x;\ \ u(0,t) = u(4,t) = 0.$

2.25 $u_t = 6u_{xx};\ u(x,0) = 25\sin 2\pi x;\ u(0,t) = u(6,t) = 0.$

Задача 3. Методом Фурье решить однородное уравнение теплопроводности с ненулевыми граничными условиями.

3.1 $u_t = 9u_{xx};\ \ u(x,0) = 5\sin 2\pi x - 1 + 3x;\ \ u(0,t) = -1;\ u(2,t) = 5.$

3.2 $u_t = 8u_{xx};\ \ u(x,0) = 6\sin 3\pi x + 2 - 3x;\ \ u(0,t) = 2;\ u(3,t) = -7.$

3.3 $u_t = 7u_{xx}$; $u(x,0) = 7\sin 2\pi x - 3 + 4x$; $u(0,t) = -3$; $u(1,t) = 1$.

3.4 $u_t = 6u_{xx}$; $u(x,0) = 8\sin 4\pi x + 4 - 5x$; $u(0,t) = 4$; $u(2,t) = -6$.

3.5 $u_t = 5u_{xx}$; $u(x,0) = 9\sin 3\pi x - 5 + 2x$; $u(0,t) = -5$; $u(3,t) = 1$.

3.6 $u_t = 9u_{xx}$; $u(x,0) = 8\sin 3\pi x + 6 - 2x$; $u(0,t) = 6$; $u(4,t) = -2$.

3.7 $u_t = 8u_{xx}$; $u(x,0) = 7\sin 2\pi x - 7 + 3x$; $u(0,t) = -7$; $u(3,t) = 2$.

3.8 $u_t = 7u_{xx}$; $u(x,0) = 6\sin 3\pi x + 8 - 3x$; $u(0,t) = 8$; $u(4,t) = -4$.

3.9 $u_t = 4u_{xx}$; $u(x,0) = 5\sin 4\pi x - 9 + 5x$; $u(0,t) = -9$; $u(2,t) = 1$.

3.10 $u_t = 3u_{xx}$; $u(x,0) = 4\sin 5\pi x + 9 - 4x$; $u(0,t) = 9$; $u(3,t) = -3$.

3.11 $u_t = 2u_{xx}$; $u(x,0) = 3\sin 6\pi x - 8 + 5x$; $u(0,t) = -8$; $u(2,t) = 2$.

3.12 $u_t = 3u_{xx}$; $u(x,0) = 2\sin 4\pi x + 7 - 5x$; $u(0,t) = 7$; $u(1,t) = 2$.

3.13 $u_t = 5u_{xx}$; $u(x,0) = 3\sin 3\pi x - 6 + 4x$; $u(0,t) = -6$; $u(3,t) = 6$.

3.14 $u_t = 6u_{xx}$; $u(x,0) = 4\sin 4\pi x + 5 - 4x$; $u(0,t) = 5$; $u(2,t) = -3$.

3.15 $u_t = 8u_{xx}$; $u(x,0) = 5\sin 2\pi x - 4 + 3x$; $u(0,t) = -4$; $u(1,t) = -1$.

3.16 $u_t = 7u_{xx}$; $u(x,0) = 6\sin 3\pi x + 3 + 2x$; $u(0,t) = 3$; $u(2,t) = 7$.

3.17 $u_t = 6u_{xx}$; $u(x,0) = 7\sin 4\pi x - 2 + x$; $u(0,t) = -2$; $u(3,t) = 1$.

3.18 $u_t = 2u_{xx}$; $u(x,0) = 8\sin 7\pi x + 1 - x$; $u(0,t) = 1$; $u(2,t) = -1$.

3.19 $u_t = 4u_{xx}$; $u(x,0) = 9\sin 3\pi x - 1 - 2x$; $u(0,t) = -1$; $u(1,t) = -3$.

3.20 $u_t = 6u_{xx}$; $u(x,0) = 8\sin 4\pi x + 3 - 4x$; $u(0,t) = 3$; $u(2,t) = -5$.

3.21 $u_t = 7u_{xx}$; $u(x,0) = 7\sin 3\pi x - 5 + 6x$; $u(0,t) = -5$; $u(1,t) = 1$.

3.22 $u_t = 8u_{xx}$; $u(x,0) = 6\sin 2\pi x + 7 - 5x$; $u(0,t) = 7$; $u(2,t) = -3$.

3.23 $u_t = 9u_{xx}$; $u(x,0) = 5\sin 3\pi x - 9 + 4x$; $u(0,t) = -9$; $u(3,t) = 3$.

3.24 $u_t = 8u_{xx}$; $u(x,0) = 4\sin 3\pi x + 8 - 3x$; $u(0,t) = 8$; $u(2,t) = 2$.

3.25 $u_t = 7u_{xx}$; $u(x,0) = 3\sin 2\pi x - 6 + 2x$; $u(0,t) = -6$; $u(3,t) = 0$.

Задача 4. Методом Фурье решить однородное уравнение колебаний с нулевыми граничными условиями.

4.1 $u_{tt} = 64u_{xx}$; $u_x(0,t) = u_x(6,t) = 0$; $u(x,0) = 0$, $u_t(x,0) = 8\pi\cos\pi x$.

4.2 $u_{tt} = 81u_{xx}$; $u_x(0,t) = u_x(5,t) = 0$; $u(x,0) = 2\cos\pi x$, $u_t(x,0) = 0$.

4.3 $u_{tt} = 36u_{xx}$; $u_x(0,t) = u_x(5,t) = 0$; $u(x,0) = 0$, $u_t(x,0) = 12\pi\cos 2\pi x$.

4.4 $u_{tt} = 49u_{xx}$; $u_x(0,t) = u_x(4,t) = 0$; $u(x,0) = 4\cos 2\pi x$, $u_t(x,0) = 0$.

4.5 $u_{tt} = 16u_{xx}$; $u_x(0,t) = u_x(4,t) = 0$; $u(x,0) = 0$, $u_t(x,0) = 12\pi\cos 3\pi x$.

4.6 $u_{tt} = 25u_{xx}$; $u_x(0,t) = u_x(3,t) = 0$; $u(x,0) = 6\cos 3\pi x$, $u_t(x,0) = 0$.

4.7 $u_{tt} = 4u_{xx}$; $u_x(0,t) = u_x(3,t) = 0$; $u(x,0) = 0$, $u_t(x,0) = 8\pi\cos 4\pi x$.

4.8 $u_{tt} = 9u_{xx}$; $u_x(0,t) = u_x(2,t) = 0$; $u(x,0) = 8\cos 4\pi x$, $u_t(x,0) = 0$.

4.9 $u_{tt} = u_{xx}$; $u_x(0,t) = u_x(3,t) = 0$; $u(x,0) = 0$, $u_t(x,0) = 5\pi\cos 5\pi x$.

4.10 $u_{tt} = u_{xx}$; $u_x(0,t) = u_x(1,t) = 0$; $u(x,0) = 10\cos 5\pi x$, $u_t(x,0) = 0$.

4.11 $u_{tt} = 9u_{xx}$; $u_x(0,t) = u_x(1,t) = 0$; $u(x,0) = 0$, $u_t(x,0) = 18\pi\cos 6\pi x$.

4.12 $u_{tt} = 4u_{xx}$; $u_x(0,t) = u_x(2,t) = 0$; $u(x,0) = 12\cos 6\pi x$, $u_t(x,0) = 0$.

4.13 $u_{tt} = 25u_{xx}$; $u_x(0,t) = u_x(2,t) = 0$; $u(x,0) = 0$, $u_t(x,0) = 25\pi\cos 5\pi x$.

4.14 $u_{tt} = 16u_{xx}$; $u_x(0,t) = u_x(3,t) = 0$; $u(x,0) = 14\cos 5\pi x$, $u_t(x,0) = 0$.

4.15 $u_{tt} = 49u_{xx}$; $u_x(0,t) = u_x(3,t) = 0$; $u(x,0) = 0$, $u_t(x,0) = 28\pi\cos 4\pi x$.

4.16 $u_{tt} = 36u_{xx}$; $u_x(0,t) = u_x(4,t) = 0$; $u(x,0) = 16\cos 4\pi x$, $u_t(x,0) = 0$.

4.17 $u_{tt} = 81u_{xx}$; $u_x(0,t) = u_x(4,t) = 0$; $u(x,0) = 0$, $u_t(x,0) = 27\pi\cos 3\pi x$.

4.18 $u_{tt} = 64u_{xx}$; $u_x(0,t) = u_x(5,t) = 0$; $u(x,0) = 18\cos 3\pi x$, $u_t(x,0) = 0$.

4.19 $u_{tt} = 4u_{xx}$; $u_x(0,t) = u_x(1,t) = 0$; $u(x,0) = 0$, $u_t(x,0) = 14\pi\cos 7\pi x$.

4.20 $u_{tt} = u_{xx}$; $u_x(0,t) = u_x(2,t) = 0$; $u(x,0) = 20\cos 7\pi x$, $u_t(x,0) = 0$.

4.21 $u_{tt} = 16u_{xx}$; $u_x(0,t) = u_x(2,t) = 0$; $u(x,0) = 0$, $u_t(x,0) = 24\pi\cos 6\pi x$.

4.22 $u_{tt} = 9u_{xx}$; $u_x(0,t) = u_x(3,t) = 0$; $u(x,0) = 22\cos 6\pi x$, $u_t(x,0) = 0$.

4.23 $u_{tt} = 36u_{xx}$; $u_x(0,t) = u_x(3,t) = 0$; $u(x,0) = 0$, $u_t(x,0) = 30\pi\cos 5\pi x$.

4.24 $u_{tt} = 25u_{xx}$; $u_x(0,t) = u_x(4,t) = 0$; $u(x,0) = 24\cos 5\pi x$, $u_t(x,0) = 0$.

4.25 $u_{tt} = 64u_{xx}$; $u_x(0,t) = u_x(4,t) = 0$; $u(x,0) = 0$, $u_t(x,0) = 32\pi\cos 4\pi x$.

Задача 5. Методом Фурье решить неоднородное волновое уравнение с нулевыми граничными и начальными условиями:

$$u(0,t) = 0;\ u(\pi,t) = 0;\ \ u(x,0) = 0;\ u_t(x,0) = 0.$$

5.1 $u_{tt} = u_{xx} + 65e^{-8t} \cdot \sin x$. 5.2 $u_{tt} = \dfrac{1}{4}u_{xx} + 3\sin 2t \cdot \sin 2x$.

5.3 $u_{tt} = u_{xx} + 16\cos 8t \cdot \sin 8x$. 5.4 $u_{tt} = \dfrac{1}{9}u_{xx} + 8\sin 3t \cdot \sin 3x$.

5.5 $u_{tt} = \dfrac{1}{16}u_{xx} + 50e^{-7t} \cdot \sin 4x$. 5.6 $u_{tt} = \dfrac{1}{25}u_{xx} + 3\cos 2t \cdot \sin 5x$.

5.7 $u_{tt} = 4u_{xx} + 28\cos 14t \cdot \sin 7x$. 5.8 $u_{tt} = \dfrac{1}{36}u_{xx} + 8\cos 3t \cdot \sin 6x$.

5.9 $u_{tt} = \dfrac{1}{49}u_{xx} + 37e^{-6t} \cdot \sin 7x$. 5.10 $u_{tt} = \dfrac{1}{64}u_{xx} + 15\sin 4t \cdot \sin 8x$.

5.11 $u_{tt} = 9u_{xx} + 36\cos 18t \cdot \sin 6x$. 5.12 $u_{tt} = \dfrac{1}{81}u_{xx} + 15\cos 4t \cdot \sin 9x$.

5.13 $u_{tt} = u_{xx} + 26e^{-5t} \cdot \sin x$. 5.14 $u_{tt} = \dfrac{1}{4}u_{xx} + 24\sin 5t \cdot \sin 2x$.

5.15 $u_{tt} = 16u_{xx} + 40\cos 20t \cdot \sin 5x$. 5.16 $u_{tt} = \dfrac{1}{9}u_{xx} + 24\cos 5t \cdot \sin 3x$.

5.17 $u_{tt} = \dfrac{1}{16}u_{xx} + 17e^{-4t} \cdot \sin 4x$. 5.18 $u_{tt} = \dfrac{1}{25}u_{xx} + 35\sin 6t \cdot \sin 5x$.

5.19 $u_{tt} = 25u_{xx} + 40\cos 20t \cdot \sin 4x$. 5.20 $u_{tt} = \dfrac{1}{36}u_{xx} + 35\cos 6t \cdot \sin 5x$.

5.21 $u_{tt} = \dfrac{1}{49}u_{xx} + 10e^{-3t} \cdot \sin 7x$. 5.22 $u_{tt} = \dfrac{1}{64}u_{xx} + 48\sin 7t \cdot \sin 8x$.

5.23 $u_{tt} = 36u_{xx} + 40\cos 18t \cdot \sin 3x$. 5.24 $u_{tt} = \dfrac{1}{81}u_{xx} + 48\cos 7t \cdot \sin 9x$.

5.25 $u_{tt} = u_{xx} + 5e^{-2t} \cdot \sin x$.

Задача 6. Методом Фурье решить уравнение колебаний прямоугольной мембраны.

6.1 $u_{tt} = \Delta u$; $u(x,y,0) = \dfrac{xy}{64}(2-x)(3-y)$; $u_t(x,y,0) = 0$;

$$u(0,y,t) = u(x,0,t) = u(2,y,t) = u(x,3,t) = 0.$$

6.2 $u_{tt} = 4\Delta u$; $u(x,y,0) = xy(3-x)(4-y)$; $u_t(x,y,0) = 0$;

$$u(0,y,t) = u(x,0,t) = u(3,y,t) = u(x,4,t) = 0.$$

6.3 $u_{tt} = 9\Delta u$; $u(x,y,0) = xy(4-x)(5-y)$; $u_t(x,y,0) = 0$;

$$u(0,y,t) = u(x,0,t) = u(4,y,t) = u(x,5,t) = 0.$$

6.4 $u_{tt} = 16\Delta u$; $u(x,y,0) = xy(5-x)(6-y)$; $u_t(x,y,0) = 0$;

$$u(0,y,t)=u(x,0,t)=u(5,y,t)=u(x,6,t)=0.$$

6.5 $u_{tt}=25\Delta u;\;\; u(x,y,0)=xy(6-x)(2-y);\; u_t(x,y,0)=0;$

$$u(0,y,t)=u(x,0,t)=u(6,y,t)=u(x,2,t)=0.$$

6.6 $u_{tt}=4\Delta u;\;\; u(x,y,0)=xy(2-x)(4-y);\; u_t(x,y,0)=0;$

$$u(0,y,t)=u(x,0,t)=u(2,y,t)=u(x,4,t)=0.$$

6.7 $u_{tt}=9\Delta u;\;\; u(x,y,0)=xy(3-x)(5-y);\; u_t(x,y,0)=0;$

$$u(0,y,t)=u(x,0,t)=u(3,y,t)=u(x,5,t)=0.$$

6.8 $u_{tt}=16\Delta u;\;\; u(x,y,0)=xy(4-x)(6-y);\; u_t(x,y,0)=0;$

$$u(0,y,t)=u(x,0,t)=u(4,y,t)=u(x,6,t)=0.$$

6.9 $u_{tt}=25\Delta u;\;\; u(x,y,0)=xy(5-x)(2-y);\; u_t(x,y,0)=0;$

$$u(0,y,t)=u(x,0,t)=u(5,y,t)=u(x,2,t)=0.$$

6.10 $u_{tt}=\Delta u;\;\; u(x,y,0)=xy(6-x)(3-y);\; u_t(x,y,0)=0;$

$$u(0,y,t)=u(x,0,t)=u(6,y,t)=u(x,3,t)=0.$$

6.11 $u_{tt}=9\Delta u;\;\; u(x,y,0)=xy(2-x)(5-y);\; u_t(x,y,0)=0;$

$$u(0,y,t)=u(x,0,t)=u(2,y,t)=u(x,5,t)=0.$$

6.12 $u_{tt}=16\Delta u;\;\; u(x,y,0)=xy(3-x)(6-y);\; u_t(x,y,0)=0;$

$$u(0,y,t)=u(x,0,t)=u(3,y,t)=u(x,6,t)=0.$$

6.13 $u_{tt}=25\Delta u;\;\; u(x,y,0)=xy(4-x)(2-y);\; u_t(x,y,0)=0;$

$$u(0,y,t)=u(x,0,t)=u(4,y,t)=u(x,2,t)=0.$$

6.14 $u_{tt}=\Delta u;\;\; u(x,y,0)=xy(5-x)(3-y);\; u_t(x,y,0)=0;$

$$u(0,y,t)=u(x,0,t)=u(5,y,t)=u(x,3,t)=0.$$

6.15 $u_{tt} = 4\Delta u;\ \ u(x,y,0) = xy(6-x)(4-y);\ u_t(x,y,0) = 0;$

$$u(0,y,t) = u(x,0,t) = u(6,y,t) = u(x,4,t) = 0.$$

6.16 $u_{tt} = 16\Delta u;\ \ u(x,y,0) = xy(2-x)(6-y);\ u_t(x,y,0) = 0;$

$$u(0,y,t) = u(x,0,t) = u(2,y,t) = u(x,6,t) = 0.$$

6.17 $u_{tt} = 25\Delta u;\ \ u(x,y,0) = xy(3-x)(2-y);\ u_t(x,y,0) = 0;$

$$u(0,y,t) = u(x,0,t) = u(3,y,t) = u(x,2,t) = 0.$$

6.18 $u_{tt} = \Delta u;\ \ u(x,y,0) = xy(4-x)(3-y);\ u_t(x,y,0) = 0;$

$$u(0,y,t) = u(x,0,t) = u(4,y,t) = u(x,3,t) = 0.$$

6.19 $u_{tt} = 4\Delta u;\ \ u(x,y,0) = xy(5-x)(4-y);\ u_t(x,y,0) = 0;$

$$u(0,y,t) = u(x,0,t) = u(5,y,t) = u(x,4,t) = 0.$$

6.20 $u_{tt} = 9\Delta u;\ \ u(x,y,0) = xy(6-x)(5-y);\ u_t(x,y,0) = 0;$

$$u(0,y,t) = u(x,0,t) = u(6,y,t) = u(x,5,t) = 0.$$

6.21 $u_{tt} = 25\Delta u;\ \ u(x,y,0) = xy(2-x)(2-y);\ u_t(x,y,0) = 0;$

$$u(0,y,t) = u(x,0,t) = u(2,y,t) = u(x,2,t) = 0.$$

6.22 $u_{tt} = \Delta u;\ \ u(x,y,0) = xy(3-x)(3-y);\ u_t(x,y,0) = 0;$

$$u(0,y,t) = u(x,0,t) = u(3,y,t) = u(x,3,t) = 0.$$

6.23 $u_{tt} = 4\Delta u;\ \ u(x,y,0) = xy(4-x)(4-y);\ u_t(x,y,0) = 0;$

$$u(0,y,t) = u(x,0,t) = u(4,y,t) = u(x,4,t) = 0.$$

6.24 $u_{tt} = 9\Delta u;\ \ u(x,y,0) = xy(5-x)(5-y);\ u_t(x,y,0) = 0;$

$$u(0,y,t) = u(x,0,t) = u(5,y,t) = u(x,5,t) = 0.$$

6.25 $u_{tt} = 16\Delta u;\ \ u(x,y,0) = xy(6-x)(6-y);\ u_t(x,y,0) = 0;$

$$u(0,y,t) = u(x,0,t) = u(6,y,t) = u(x,6,t) = 0.$$

Задача 7. Решить задачу Дирихле для уравнения Лапласа

$$\Delta u = 0$$

в круге

$$0 < r < 1, 0 \leq \varphi < 2\pi,$$

где

$$r, \varphi$$

- полярные координаты, на границе которого искомая функция

$$u(r,\varphi)$$

имеет следующие значения:

$$7.1 \;\; u(1,\varphi) = \cos 9\varphi. \quad 7.2 \;\; u(1,\varphi) = 2\sin 8\varphi.$$

$$7.3 \;\; u(1,\varphi) = 3\cos 7\varphi. \quad 7.4 \;\; u(1,\varphi) = 4\sin 6\varphi.$$

$$7.5 \;\; u(1,\varphi) = 5\cos 5\varphi. \quad 7.6 \;\; u(1,\varphi) = 6\sin 4\varphi.$$

$$7.7 \;\; u(1,\varphi) = 7\cos 3\varphi. \quad 7.8 \;\; u(1,\varphi) = 8\sin 2\varphi.$$

$$7.9 \;\; u(1,\varphi) = 9\cos 2\varphi. \quad 7.10 \;\; u(1,\varphi) = 10\sin 3\varphi.$$

$$7.11 \;\; u(1,\varphi) = 11\cos 4\varphi. \quad 7.12 \;\; u(1,\varphi) = 12\sin 5\varphi.$$

$$7.13 \;\; u(1,\varphi) = 13\cos 6\varphi. \quad 7.14 \;\; u(1,\varphi) = 14\sin 7\varphi.$$

$$7.15 \;\; u(1,\varphi) = 15\cos 8\varphi. \quad 7.16 \;\; u(1,\varphi) = 16\sin 9\varphi.$$

$$7.17 \;\; u(1,\varphi) = 17\cos 9\varphi. \quad 7.18 \;\; u(1,\varphi) = 18\sin 8\varphi.$$

$$7.19\ \ u(1,\varphi) = 19\cos 7\varphi. \quad 7.20\ \ u(1,\varphi) = 20\sin 6\varphi.$$

$$7.21\ \ u(1,\varphi) = 21\cos 5\varphi. \quad 7.22\ \ u(1,\varphi) = 22\sin 4\varphi.$$

$$7.23\ \ u(1,\varphi) = 23\cos 3\varphi. \quad 7.24\ \ u(1,\varphi) = 24\sin 2\varphi.$$

$$7.25\ \ u(1,\varphi) = 25\cos 2\varphi.$$

Задача 8. Найти решение уравнения Лапласа

$$\Delta u = 0$$

в круговом секторе

$$0 < r < 1,\ \ 0 < \varphi < \alpha,$$

(

$$r, \varphi$$

- полярные координаты,

$$\alpha < 2\pi$$

), на границе которого искомая функция

$$u(r,\varphi)$$

удовлетворяет следующим условиям:

$$8.1\ \ u(1,\varphi) = \sin 6\varphi;\ \ u(r,0) = u(r,\pi/3) = 0.$$

$$8.2\ \ u(1,\varphi) = 2\cos 2\varphi;\ \ u_\varphi(r,0) = u_\varphi(r,\pi) = 0.$$

$$8.3\ \ u(1,\varphi) = 3\cos 15\varphi;\ \ u_\varphi(r,0) = 0,\ u(r,\pi/6) = 0.$$

8.4 $u(1, \varphi) = 4 \sin 14\varphi; \quad u(r, 0) = 0, \; u_\varphi(r, \pi/4) = 0.$

8.5 $u(1, \varphi) = 5 \sin 3\varphi; \quad u(r, 0) = u(r, 2\pi/3) = 0.$

8.6 $u(1, \varphi) = 6 \cos 6\varphi; \quad u_\varphi(r, 0) = u_\varphi(r, 7\pi/6) = 0.$

8.7 $u(1, \varphi) = 7 \cos 10\varphi; \quad u_\varphi(r, 0) = 0, \; u(r, \pi/4) = 0.$

8.8 $u(1, \varphi) = 8 \sin 7\varphi; \quad u(r, 0) = 0, \; u_\varphi(r, \pi/2) = 0.$

8.9 $u(1, \varphi) = 9 \sin 4\varphi; \quad u(r, 0) = u(r, 3\pi/4) = 0.$

8.10 $u(1, \varphi) = 10 \cos 4\varphi; \quad u_\varphi(r, 0) = u_\varphi(r, 5\pi/4) = 0.$

8.11 $u(1, \varphi) = 11 \cos 5\varphi; \quad u_\varphi(r, 0) = 0, \; u(r, \pi/2) = 0.$

8.12 $u(1, \varphi) = 12 \sin 3\varphi; \quad u(r, 0) = 0, \; u_\varphi(r, 3\pi/2) = 0.$

8.13 $u(1, \varphi) = 13 \sin 6\varphi; \quad u(r, 0) = u(r, 5\pi/6) = 0.$

8.14 $u(1, \varphi) = 10 \cos 3\varphi; \quad u_\varphi(r, 0) = u_\varphi(r, 4\pi/3) = 0.$

8.15 $u(1, \varphi) = 15 \cos \varphi; \quad u_\varphi(r, 0) = 0, \; u(r, 3\pi/2) = 0.$

8.16 $u(1, \varphi) = 16 \sin 21\varphi; \quad u(r, 0) = 0, \; u_\varphi(r, \pi/6) = 0.$

8.17 $u(1, \varphi) = 17 \sin 9\varphi; \quad u(r, 0) = u(r, \pi/3) = 0.$

8.18 $u(1, \varphi) = 18 \cos 4\varphi; \quad u_\varphi(r, 0) = u_\varphi(r, \pi) = 0.$

8.19 $u(1, \varphi) = 19 \cos 21\varphi; \quad u_\varphi(r, 0) = 0, \; u(r, \pi/6) = 0.$

8.20 $u(1, \varphi) = 20 \sin 15\varphi; \quad u(r, 0) = 0, \; u_\varphi(r, \pi/6) = 0.$

8.21 $u(1, \varphi) = 21 \sin 6\varphi; \quad u(r, 0) = u(r, 2\pi/3) = 0.$

8.22 $u(1, \varphi) = 22 \cos 12\varphi; \quad u_\varphi(r, 0) = u_\varphi(r, \pi/3) = 0.$

8.23 $u(1, \varphi) = 23 \cos 14\varphi; \quad u_\varphi(r, 0) = 0, \; u(r, \pi/4) = 0.$

8.24 $u(1, \varphi) = 4 \sin 10\varphi; \quad u(r, 0) = 0, \; u_\varphi(r, \pi/4) = 0.$

8.25 $u(1,\varphi) = 25\sin 3\varphi;\ \ u(r,0) = u(r,\pi) = 0.$

Задача 9. Применяя интегральное преобразование Фурье, решить задачу Коши для уравнения теплопроводности.

9.1 $u_t = u_{xx};\ \ u(x,0) = e^{-x^2+x}.$ 9.2 $u_t = 2u_{xx};\ \ u(x,0) = e^{-x^2}.$

9.3 $u_t = 3u_{xx};\ \ u(x,0) = e^{-2x^2}.$ 9.4 $u_t = 4u_{xx};\ \ u(x,0) = e^{-2x^2+x}.$

9.5 $u_t = 5u_{xx};\ \ u(x,0) = e^{-2x^2-x}.$ 9.6 $u_t = 6u_{xx};\ \ u(x,0) = e^{-x^2-x}.$

9.7 $u_t = 7u_{xx};\ \ u(x,0) = e^{-2x^2+2x}.$ 9.8 $u_t = 8u_{xx};\ \ u(x,0) = e^{-3x^2}.$

9.9 $u_t = 9u_{xx};\ \ u(x,0) = e^{-3x^2+x}.$ 9.10 $u_t = 10u_{xx};\ \ u(x,0) = e^{-2x^2-2x}.$

9.11 $u_t = 11u_{xx};\ \ u(x,0) = e^{-3x^2-x}.$ 9.12 $u_t = 12u_{xx};\ \ u(x,0) = e^{-4x^2}.$

9.13 $u_t = 13u_{xx};\ \ u(x,0) = e^{-3x^2+2x}.$ 9.14 $u_t = 14u_{xx};\ \ u(x,0) = e^{-4x^2+x}.$

9.15 $u_t = 15u_{xx};\ \ u(x,0) = e^{-3x^2-2x}.$ 9.16 $u_t = 16u_{xx};\ \ u(x,0) = e^{-4x^2-2x}.$

9.17 $u_t = 15u_{xx};\ \ u(x,0) = e^{-x^2+2x}.$ 9.18 $u_t = 14u_{xx};\ \ u(x,0) = e^{-3x^2+3x}.$

9.19 $u_t = 13u_{xx};\ \ u(x,0) = e^{-2x^2+4x}.$ 9.20 $u_t = 12u_{xx};\ \ u(x,0) = e^{-x^2-2x}.$

9.21 $u_t = 11u_{xx};\ \ u(x,0) = e^{-3x^2-3x}.$ 9.22 $u_t = 10u_{xx};\ \ u(x,0) = e^{-2x^2-4x}.$

9.23 $u_t = 9u_{xx};\ \ u(x,0) = e^{-x^2+4x}.$ 9.24 $u_t = 8u_{xx};\ \ u(x,0) = e^{-x^2-4x}.$

9.25 $u_t = 7u_{xx};\ \ u(x,0) = e^{-4x^2+2x}.$

Задача 10. Найти общее решение уравнения.

10.1 $y'' + \frac{1}{x}y' + (1^2 - \frac{25^2}{x^2})y = 0.$ 10.2 $y'' + \frac{1}{x}y' + (2^2 - \frac{24^2}{x^2})y = 0.$

10.3 $y'' + \frac{1}{x}y' + (3^2 - \frac{23^2}{x^2})y = 0.$ 10.4 $y'' + \frac{1}{x}y' + (4^2 - \frac{22^2}{x^2})y = 0.$

10.5 $y'' + \frac{1}{x}y' + (5^2 - \frac{21^2}{x^2})y = 0.$ 10.6 $y'' + \frac{1}{x}y' + (6^2 - \frac{20^2}{x^2})y = 0.$

10.7 $y'' + \frac{1}{x}y' + (7^2 - \frac{19^2}{x^2})y = 0.$ 10.8 $y'' + \frac{1}{x}y' + (8^2 - \frac{18^2}{x^2})y = 0.$

10.9 $y'' + \frac{1}{x}y' + (9^2 - \frac{17^2}{x^2})y = 0.$ 10.10 $y'' + \frac{1}{x}y' + (10^2 - \frac{16^2}{x^2})y = 0.$

10.11 $y'' + \frac{1}{x}y' + (11^2 - \frac{15^2}{x^2})y = 0.$ 10.12 $y'' + \frac{1}{x}y' + (12^2 - \frac{14^2}{x^2})y = 0.$

10.13 $y'' + \frac{1}{x}y' + (13^2 - \frac{13^2}{x^2})y = 0.$ 10.14 $y'' + \frac{1}{x}y' + (14^2 - \frac{12^2}{x^2})y = 0.$

10.15 $y'' + \frac{1}{x}y' + (15^2 - \frac{11^2}{x^2})y = 0.$ 10.16 $y'' + \frac{1}{x}y' + (16^2 - \frac{10^2}{x^2})y = 0.$

10.17 $y'' + \frac{1}{x}y' + (17^2 - \frac{9^2}{x^2})y = 0.$ 10.18 $y'' + \frac{1}{x}y' + (18^2 - \frac{8^2}{x^2})y = 0.$

10.19 $y'' + \frac{1}{x}y' + (19^2 - \frac{7^2}{x^2})y = 0.$ 10.20 $y'' + \frac{1}{x}y' + (20^2 - \frac{6^2}{x^2})y = 0.$

10.21 $y'' + \frac{1}{x}y' + (21^2 - \frac{5^2}{x^2})y = 0.$ 10.22 $y'' + \frac{1}{x}y' + (22^2 - \frac{4^2}{x^2})y = 0.$

10.23 $y'' + \frac{1}{x}y' + (23^2 - \frac{3^2}{x^2})y = 0.$ 10.24 $y'' + \frac{1}{x}y' + (24^2 - \frac{2^2}{x^2})y = 0.$

10.25 $y'' + \frac{1}{x}y' + (25^2 - \frac{1^2}{x^2})y = 0.$

Задача 11. Разложить в ряд Фурье - Бесселя функцию

$$f(x) = x^p,$$

по системе функций

$$J_p(\mu_i x)$$

в интервале

$$0 < x < 1,$$

где

$$\mu_i$$

- положительные корни уравнения

$$J_p(x) = 0.$$

11.1 $f(x) = x^{-1/2}, (p = -1/2)$. 11.2 $f(x) = 1, (p = 0)$. 11.3 $f(x) = x^{1/2}, (p = 1/2)$.

11.4 $f(x) = x, (p = 1)$. 11.5 $f(x) = x^{3/2}, (p = 3/2)$. 11.6 $f(x) = x^2, (p = 2)$.

11.7 $f(x) = x^{5/2}, (p = 5/2)$. 11.8 $f(x) = x^3, (p = 3)$. 11.9 $f(x) = x^{7/2}, (p = 7/2)$.

11.10 $f(x) = x^4, (p = 4)$. 11.11 $f(x) = x^{9/2}, (p = 9/2)$. 11.12 $f(x) = x^5, (p = 5)$.

11.13 $f(x) = x^{11/2}, (p = 11/2)$. 11.14 $f(x) = x^{-1/4}, (p = -1/4)$.

11.15 $f(x) = x^{-1/3}, (p = -1/3)$. 11.16 $f(x) = x^{1/4}, (p = 1/4)$.

11.17 $f(x) = x^{3/4}, (p = 3/4)$. 11.18 $f(x) = x^{5/4}, (p = 5/4)$.

11.19 $f(x) = x^{7/4}, (p = 7/4)$. 11.20 $f(x) = x^{9/4}, P = 9/4)$.

11.21 $f(x) = x^{11/4}, (p = 11/4)$. 11.22 $f(x) = x^6, (p = 6)$.

11.23 $f(x) = x^{1/3}, (p = 1/3)$. 11.24 $f(x) = x^{2/3}, (p = 2/3)$.

11.25 $f(x) = x^{4/3}, (p = 4/3)$.

Задача 12. Методом Фурье решить уравнение колебаний круглой мембраны.

12.1 $u_{tt} = \Delta u, \;\; 0 \le r < 25, \;\; 0 < t < \infty;$

$$u(r,0) = \frac{1}{8}\left[1 - \left(\frac{r}{25}\right)^2\right], \;\; u_t(r,0) = 0, \; u(25,t) = 0.$$

12.2 $u_{tt} = 2\Delta u, \;\; 0 \le r < 24, \;\; 0 < t < \infty;$

$$u(r,0) = \frac{1}{8}\left[1 - \left(\frac{r}{24}\right)^2\right], \;\; u_t(r,0) = 0, \; u(24,t) = 0.$$

12.3 $u_{tt} = 3\Delta u, \;\; 0 \le r < 23, \;\; 0 < t < \infty;$

$$u(r,0) = \frac{1}{8}\left[1 - \left(\frac{r}{23}\right)^2\right], \;\; u_t(r,0) = 0, \; u(23,t) = 0.$$

12.4 $u_{tt} = 4\Delta u, \;\; 0 \le r < 22, \;\; 0 < t < \infty;$

$$u(r,0) = \frac{1}{8}\left[1 - \left(\frac{r}{22}\right)^2\right], \;\; u_t(r,0) = 0, \; u(22,t) = 0.$$

12.5 $u_{tt} = 5\Delta u, \;\; 0 \le r < 21, \;\; 0 < t < \infty;$

$$u(r,0) = \frac{1}{8}\left[1 - \left(\frac{r}{21}\right)^2\right], \;\; u_t(r,0) = 0, \; u(21,t) = 0.$$

12.6 $u_{tt} = 6\Delta u, \;\; 0 \le r < 20, \;\; 0 < t < \infty;$

$$u(r,0) = \frac{1}{8}\left[1 - \left(\frac{r}{20}\right)^2\right], \;\; u_t(r,0) = 0, \; u(20,t) = 0.$$

12.7 $u_{tt} = 7\Delta u, \;\; 0 \le r < 19, \;\; 0 < t < \infty;$

$$u(r,0) = \frac{1}{8}\left[1 - \left(\frac{r}{19}\right)^2\right], \;\; u_t(r,0) = 0, \; u(19,t) = 0.$$

12.8 $u_{tt} = 8\Delta u, \;\; 0 \le r < 18, \;\; 0 < t < \infty;$

$$u(r,0) = \frac{1}{8}\left[1 - \left(\frac{r}{18}\right)^2\right], \;\; u_t(r,0) = 0, \; u(18,t) = 0.$$

12.9 $u_{tt} = 9\Delta u, \;\; 0 \le r < 17, \;\; 0 < t < \infty;$

$$u(r,0) = \frac{1}{8}\left[1 - \left(\frac{r}{17}\right)^2\right], \;\; u_t(r,0) = 0, \; u(17,t) = 0.$$

12.10 $u_{tt} = 10\Delta u, \;\; 0 \le r < 16, \;\; 0 < t < \infty;$

$$u(r,0) = \frac{1}{8}\left[1 - \left(\frac{r}{16}\right)^2\right], \;\; u_t(r,0) = 0, \; u(16,t) = 0.$$

12.11 $u_{tt} = 11\Delta u, \;\; 0 \le r < 15, \;\; 0 < t < \infty;$

$$u(r,0) = \frac{1}{8}\left[1 - \left(\frac{r}{15}\right)^2\right], \;\; u_t(r,0) = 0, \; u(15,t) = 0.$$

12.12 $u_{tt} = 12\Delta u, \;\; 0 \le r < 14, \;\; 0 < t < \infty;$

$$u(r,0) = \frac{1}{8}\left[1 - \left(\frac{r}{14}\right)^2\right], \;\; u_t(r,0) = 0, \; u(14,t) = 0.$$

12.13 $u_{tt} = 13\Delta u, \;\; 0 \le r < 13, \;\; 0 < t < \infty;$

$$u(r,0) = \frac{1}{8}\left[1 - \left(\frac{r}{13}\right)^2\right], \;\; u_t(r,0) = 0, \; u(13,t) = 0.$$

12.14 $u_{tt} = 14\Delta u, \;\; 0 \le r < 12, \;\; 0 < t < \infty;$

$$u(r,0) = \frac{1}{8}\left[1 - \left(\frac{r}{12}\right)^2\right], \;\; u_t(r,0) = 0, \; u(12,t) = 0.$$

12.15 $u_{tt} = 15\Delta u, \;\; 0 \le r < 11, \;\; 0 < t < \infty;$

$$u(r,0) = \frac{1}{8}\left[1 - \left(\frac{r}{11}\right)^2\right], \;\; u_t(r,0) = 0, \; u(11,t) = 0.$$

12.16 $u_{tt} = 16\Delta u, \;\; 0 \le r < 10, \;\; 0 < t < \infty;$

$$u(r,0) = \frac{1}{8}\left[1 - \left(\frac{r}{10}\right)^2\right], \;\; u_t(r,0) = 0, \; u(10,t) = 0.$$

12.17 $u_{tt} = 17\Delta u, \;\; 0 \le r < 9, \;\; 0 < t < \infty;$

$$u(r,0) = \frac{1}{8}\left[1 - \left(\frac{r}{9}\right)^2\right], \;\; u_t(r,0) = 0, \; u(9,t) = 0.$$

12.18 $u_{tt} = 18\Delta u, \quad 0 \le r < 8, \quad 0 < t < \infty;$

$$u(r,0) = \frac{1}{8}\left[1 - \left(\frac{r}{8}\right)^2\right], \quad u_t(r,0) = 0, \; u(8,t) = 0.$$

12.19 $u_{tt} = 19\Delta u, \quad 0 \le r < 7, \quad 0 < t < \infty;$

$$u(r,0) = \frac{1}{8}\left[1 - \left(\frac{r}{7}\right)^2\right], \quad u_t(r,0) = 0, \; u(7,t) = 0.$$

12.20 $u_{tt} = 20\Delta u, \quad 0 \le r < 6, \quad 0 < t < \infty;$

$$u(r,0) = \frac{1}{8}\left[1 - \left(\frac{r}{6}\right)^2\right], \quad u_t(r,0) = 0, \; u(6,t) = 0.$$

12.21 $u_{tt} = 21\Delta u, \quad 0 \le r < 5, \quad 0 < t < \infty;$

$$u(r,0) = \frac{1}{8}\left[1 - \left(\frac{r}{5}\right)^2\right], \quad u_t(r,0) = 0, \; u(5,t) = 0.$$

12.22 $u_{tt} = 22\Delta u, \quad 0 \le r < 4, \quad 0 < t < \infty;$

$$u(r,0) = \frac{1}{8}\left[1 - \left(\frac{r}{4}\right)^2\right], \quad u_t(r,0) = 0, \; u(4,t) = 0.$$

12.23 $u_{tt} = 23\Delta u, \quad 0 \le r < 3, \quad 0 < t < \infty;$

$$u(r,0) = \frac{1}{8}\left[1 - \left(\frac{r}{3}\right)^2\right], \quad u_t(r,0) = 0, \; u(3,t) = 0.$$

12.24 $u_{tt} = 24\Delta u, \quad 0 \le r < 2, \quad 0 < t < \infty;$

$$u(r,0) = \frac{1}{8}\left[1 - \left(\frac{r}{2}\right)^2\right], \quad u_t(r,0) = 0, \; u(2,t) = 0.$$

12.25 $u_{tt} = 25\Delta u, \quad 0 \le r < 1, \quad 0 < t < \infty;$

$$u(r,0) = \frac{1}{8}\left[1 - r^2\right], \quad u_t(r,0) = 0, \; u(1,t) = 0.$$